Exercise 2.1

Question 1:

Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$

(ii) $y^2 + \sqrt{2}$

(iii) $3\sqrt{t} + t\sqrt{2}$

(iv) $y + \frac{2}{y}$

(v) $y + 2y^{-1}$

Solution 1:

i) $4x^2 - 3x + 7$

One variable is involved in given polynomial which is 'x'

Therefore, it is a polynomial in one variable 'x'.

(ii) $y^2 + \sqrt{2}$

One variable is involved in given polynomial which is 'y'

Therefore, it is a polynomial in one variable 'y'.

(iii) $3\sqrt{t} + t\sqrt{2}$

No. It can be observed that the exponent of variable t in term $3\sqrt{t}$ is $\frac{1}{2}$, which is nota whole number. Therefore, this expression is not a polynomial.

(iv) $y + \frac{2}{y}$

$= y + 2y^{-1}$

The power of variable 'y' is -1 which is not a whole number.

Therefore, it is not a polynomial in one variable

No. It can be observed that the exponent of variable y in term $\frac{2}{y}$ is -1, which is not a whole number. Therefore, this expression is not a polynomial.

(v) $x^{10} + y^3 + t^{50}$

In the given expression there are 3 variables which are 'x, y, t' involved.

Therefore, it is not a polynomial in one variable.

Question 2:

Write the coefficients of x^2 in each of the following:

(i) $2+x^2+x$

(ii) $2-x^2+x^3$

(iii) $\frac{\pi}{2}x^2+x$

(iv) $\sqrt{2}x-1$

Solution 2:

(i) $2+x^2+x^3$

$=2+1(x^2)+x$

The coefficient of x^2 is 1.

(ii) $2-x^2+x^3$

$=2-1(x^2)+x$

The coefficient of x^2 is −1.

(iii) $\frac{\pi}{2}x^2+x$

The coefficient x^2 of is $\frac{\pi}{2}$.

(iv) $\sqrt{2}x-1=0x^2+\sqrt{2}x-1$

The coefficient of x^2 is 0.

Question 3:

Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Solution 3 :

Binomial of degree 35 means a polynomial is having

1. Two terms
2. Highest degree is 35

Example: $x^{35}+x^{34}$

Monomial of degree 100 means a polynomial is having

1. One term
2. Highest degree is 100

   Example : x^{100}.

Question 4:

Write the degree of each of the following polynomials:

(i) $5x^3+4x^2+7x$

(ii) $4-y^2$

(iii) $5t-\sqrt{7}$

(iv) 3

Solution 4:

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) $5x^3+4x^2+7x$

Highest power of variable ‘x’ is 3. Therefore, the degree of this polynomial is 3

(ii) $4-y^2$

Highest power of variable ‘y’ is 2. Therefore, the degree of this polynomial is 2.

(iii) $5t-\sqrt{7}$

Highest power of variable ‘t’ is 1. Therefore, the degree of this polynomial is 1.

(iv) 3

This is a constant polynomial. Degree of a constant polynomial is always 0.

Question 5: Classify the following as linear, quadratic and cubic polynomial:

(i) x^2+x

(ii) $x-x^3$

(iii) $y+y^2+4$

(iv) $1+x$
(v) 3t
(vi) r^2
(vii) $7x^2 \quad 7x^3$

Solution 5:
Linear polynomial – whose variable power is ‘1’

Quadratic polynomial - whose variable highest power is ‘2’
Cubic polynomial- whose variable highest power is ‘3’

(i) x^2+x is a quadratic polynomial as its highest degree is 2.
(ii) $x-x^3$ is a cubic polynomial as its highest degree is 3.
(iii) $y+y^2+4$ is a quadratic polynomial as its highest degree is 2.
(iv) 1 + x is a linear polynomial as its degree is 1.
(v) 3t is a linear polynomial as its degree is 1.
(vi) r^2 is a quadratic polynomial as its degree is 2.
(vii) $7x^2 \quad 7x^3$ is a cubic polynomial as highest its degree is 3.

Exercise 2.2

Question 1:
Find the value of the polynomial at $5x-4x^2+3$ at

(i) x = 0
(ii) x = −1
(iii) x = 2

Solution 1:

(i) $p(x)=5x-4x^2+3$

$p(0)=5(0)-4(0)^2+3 = 3$

(ii) $p(x)=5x-4x^2+3$

$p(-1)=5(-1)-4(-1)^2+3$
$= -5-4(1)+3=-6$

(iii) $p(x)=5x-4x^2+3$

$p(2)=5(2)-4(2)^2+3 = 10-16+3=-3$

Question 2:

Find p(0), p(1) and p(2) for each of the following polynomials:

(i) $p(y) = y^2 - y + 1$
(ii) $p(t) = 2 + t + 2t^2 - t3$
(iii) $p(x) = x^3$
(iv) $p(x) = (x - 1)(x + 1)$

Solution 2:

(i) $p(y) = y^2 - y + 1$

- $p(0) = (0)^2 - (0) + 1 = 1$
- $p(1) = (1)^2 - (1) + 1 = 1 - 1 + 1 = 1$
- $p(2) = (2)^2 - (2) + 1 = 4 - 2 + 1 = 3$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

- $p(0) = 2 + 0 + 2(0)^2 - (0)^3 = 2$
- $p(1) = 2 + (1) + 2(1)^2 - (1)^3 = 2 + 1 + 2 - 1 = 4$
- $p(2) = 2 + 2 + 2(2)^2 - (2)^3$

$= 2 + 2 + 8 - 8 = 4$

(iii) $p(x) = x^3$

- $p(0) = (0)^3 = 0$
- $p(1) = (1)^3 = 1$
- $p(2) = (2)^3 = 8$

(v) $p(x) = (x - 1)(x + 1)$

- $p(0) = (0 - 1)(0 + 1) = (-1)(1) = -1$
- $p(1) = (1 - 1)(1 + 1) = 0(2) = 0$
- $p(2) = (2 - 1)(2 + 1) = 1(3) = 3$

Question 3:

Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x)=3x+1, x=-\frac{1}{3}$

(ii) $p(x)=5x-\pi, x=\frac{4}{5}$

(iii) $p(x)=x^2-1, x=1,-1$

(iv) $p(x)=(x+1)(x-2), x=-1,2$

(v) $p(x)=x^2, x=0$

(vi) $p(x)=lm+m, x=-\frac{m}{l}$

(vii) $p(x)=3x^2-1, x=-\frac{1}{\sqrt{3}},\frac{2}{\sqrt{3}}$

(viii) $p(x)=2x+1, x=\frac{1}{2}$

Solution 3:

(i) If $x=-\frac{1}{3}$ is a zero of given polynomial p(x) = 3x + 1, then $p\left(-\frac{1}{3}\right)$ should be 0.

Here, $p\left(-\frac{1}{3}\right)=3\left(-\frac{1}{3}\right)+1 = -1+1=0$

Therefore, is a zero of the given polynomial.

(ii) If $x=\frac{4}{5}$ is a zero of polynomial p(x) = 5x − π, then $p\left(\frac{4}{5}\right)$ should be 0.

Here, $p\left(\frac{4}{5}\right)=5\left(\frac{4}{5}\right)-\pi = 4-\pi$

As $p\left(\frac{4}{5}\right) \neq 0$

Therefore, $x=\frac{4}{5}$ is not a zero of the given polynomial.

(iii) If $x = 1$ and $x = -1$ are zeroes of polynomial $p(x) = x^2 - 1$, then $p(1)$ and $p(-1)$ should be 0.

Here, $p(1) = (1)^2 - 1 = 0$, and

$p(-1) = (-1)^2 - 1 = 0$

Hence, $x = 1$ and -1 are zeroes of the given polynomial.

(iv) If $x = -1$ and $x = 2$ are zeroes of polynomial $p(x) = (x + 1)(x - 2)$, then $p(-1)$ and $p(2)$ should be 0.

Here, $p(-1) = (-1 + 1)(-1 - 2) = 0(-3) = 0$, and

$p(2) = (2 + 1)(2 - 2) = 3(0) = 0$

Therefore, $x = -1$ and $x = 2$ are zeroes of the given polynomial.

(v) If $x = 0$ is a zero of polynomial $p(x) = x^2$, then $p(0)$ should be zero.

Here, $p(0) = (0)^2 = 0$

Hence, $x = 0$ is a zero of the given polynomial.

(vi) If $p\left(\frac{-m}{l}\right)$ is a zero of polynomial $p(x) = lx + m$, then $p\left(\frac{-m}{l}\right)$ should be 0.

Here, $p\left(\frac{-m}{l}\right) = l\left(\frac{-m}{l}\right) + m = -m + m = 0$

Therefore, $x = \frac{-m}{l}$ is a zero of the given polynomial.

(vii) If $x = \frac{-1}{\sqrt{3}}$ and $x = \frac{2}{\sqrt{3}}$ are zeroes of polynomial $p(x) = 3x^2 - 1$, then

$p\left(\frac{-1}{\sqrt{3}}\right)$ and $p\left(\frac{2}{\sqrt{3}}\right)$ should be 0.

Here, $p\left(\frac{-1}{\sqrt{3}}\right)=3\left(\frac{-1}{\sqrt{3}}\right)^2-1=3\left(\frac{1}{3}\right)-1=1-1=0$, and

$p\left(\frac{2}{\sqrt{3}}\right)=3\left(\frac{2}{\sqrt{3}}\right)^2-1=3\left(\frac{4}{3}\right)-1=4-1=3$

Hence, $x=\frac{-1}{\sqrt{3}}$ is a zero of the given polynomial.

However, $x=\frac{2}{\sqrt{3}}$ is not a zero of the given polynomial.

(viii) If $x=\frac{1}{2}$ is a zero of polynomial p(x) = 2x + 1, then $p\left(\frac{1}{2}\right)$ should be 0.

Here, $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 1 + 1 = 2$

As $p\left(\frac{1}{2}\right)\neq 0$,

Therefore, $x=\frac{1}{2}$ is not a zero of the given polynomial.

Question 4:

Find the zero of the polynomial in each of the following cases:

(i) p(x) = x + 5

(ii) p(x) = x – 5

(iii) p(x) = 2x + 5

(iv) p(x) = 3x – 2

(v) p(x) = 3x

(vi) p(x) = ax, a ≠ 0

(vii) p(x) = cx + d, c ≠ 0, c, d are real numbers.

Solution 4:

Zero of a polynomial is that value of the variable at which the value of thepolynomial is obtained as 0.

(i) p(x) = x + 5
Let p(x) = 0
x + 5 = 0
x = − 5
Therefore, for x = −5, the value of the polynomial is 0 and hence, x = −5 is a zero of the given polynomial.

(ii) p(x) = x − 5
Let p(x) = 0
x − 5 = 0
x = 5
Therefore, for x = 5, the value of the polynomial is 0 and hence, x = 5 is a zero of the given polynomial.

(iii) p(x) = 2x + 5
Let p(x) = 0
2x + 5 = 0
2x = − 5

$$x = -\frac{5}{2}$$

Therefore, for $x = -\frac{5}{2}$, the value of the polynomial is 0 and hence, $x = -\frac{5}{2}$ is a zero of the given polynomial.

(iv) p(x) = 3x – 2

p(x) = 0
3x − 2 = 0
Therefore, for $x = \frac{2}{3}$, the value of the polynomial is 0 and hence, $x = \frac{2}{3}$ is a zero of the given polynomial.

(v) p(x) = 3x
Let p(x) = 0
3x = 0
x = 0
Therefore, for x = 0, the value of the polynomial is 0 and hence, x = 0 is a zero of the given polynomial.

(vi) p(x) = ax
Let p(x) = 0
ax = 0
x = 0
Therefore, for x = 0, the value of the polynomial is 0 and hence, x = 0 is a zero of the given polynomial.

(vii) p(x) = cx + d
Let p(x) = 0
cx + d = 0

$$x = \frac{-d}{c}$$

Therefore, for $x = \frac{-d}{c}$, the value of the polynomial is 0 and hence, $x = \frac{-d}{c}$ is a zero of the given polynomial.

Exercise 2.3

Question 1:

Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by

(i) $x + 1$

(ii) $x - \frac{1}{2}$

(iii) x

(iv) $x + \pi$

(v) $5 + 2x$

Solution 1:

(i) $x^3 + 3x^2 + 3x + 1 \div x + 1$

By long division, we get

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
\hline
x+1 & x^3 + 3x^2 + 3x + 1 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & \quad 2x^2 + 3x + 1 \\
 & \quad 2x^2 + 2x \\
 & \quad - \quad - \\
\hline
 & \qquad x + 1 \\
 & \qquad x + 1 \\
 & \qquad - \quad - \\
\hline
 & \qquad 0 \\
\hline
\end{array}
$$

Therefore, the remainder is 0.

(ii) $x^3 + 3x^2 + 3x + 1 \div x - \frac{1}{2}$

By long division,

$$
\begin{array}{r|l}
 & x^2 + \frac{7}{2}x + \frac{19}{4} \\
x - \frac{1}{2} & x^3 + 3x^2 + 3x + 1 \\
 & x^3 - \frac{x^2}{2} \\
 & - \quad + \\
\hline
 & \frac{7}{2}x^2 + 3x + 1 \\
 & \frac{7}{2}x^2 - \frac{7}{4}x \\
 & - \quad + \\
\hline
 & \frac{19}{4}x + 1 \\
 & \frac{19}{4}x - \frac{19}{8} \\
 & - \quad + \\
\hline
 & \frac{27}{8} \\
\hline
\end{array}
$$

Therefore, the remainder is $\frac{27}{8}$.

(iii) $x^3 + 3x^2 + 3x + 1 \div x$

By long division,

$$
\begin{array}{r|l}
 & x^2 + 3x + 3 \\
x & x^3 + 3x^2 + 3x + 1 \\
 & x^3 \\
 & - \\
\hline
 & 3x^2 + 3x + 1 \\
 & 3x^2 \\
 & - \\
\hline
 & 3x + 1 \\
 & 3x \\
 & - \\
\hline
 & 1 \\
\hline
\end{array}
$$

Therefore, the remainder is 1.

(iv) $x^3 + 3x^2 + 3x + 1 \div x + \pi$

By long division, we get

$$x^2+(3-\pi)x+(3-3\pi+\pi^2)$$

$$x+\pi\,\big)\; x^3+3x^2+3x+1$$

$$x^3+\pi x^2$$

$$- \quad -$$

$$(3-\pi)x^2+3x+1$$

$$(3-\pi)x^2+(3-\pi)\pi x$$

$$- \qquad -$$

$$\left[3-3\pi+\pi^2\right]x+1$$

$$\left[3-3\pi+\pi^2\right]x+(3-3\pi+\pi^2)\pi$$

$$- \qquad -$$

$$\left[1-3\pi+3\pi^2-\pi^3\right]$$

Therefore, the remainder is $-\pi^3+3\pi^2-3\pi+1$.

(v) 5 + 2x

By long division, we get

$$
\begin{array}{r}
\frac{x^2}{2}+\frac{x}{4}+\frac{7}{8} \\
2x+5\overline{\smash{)}\,x^3+3x^2+3x+1} \\
x^3+\frac{5}{2}x^2 \\
-\quad - \\
\hline
\frac{x^2}{2}+3x+1 \\
\frac{x^2}{2}+\frac{5x}{4} \\
-\quad - \\
\hline
\frac{7x}{4}+1 \\
\frac{7}{4}x+\frac{35}{8} \\
-\quad - \\
\hline
-\frac{27}{8} \\
\hline
\end{array}
$$

Therefore, the remainder is $-\frac{27}{8}$.

Question 2:

Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Solution 2:

$x^3 - ax^2 + 6x - a \div x - a$

By long division,

$$
\begin{array}{r}
x^2+6 \\
x-a\overline{\smash{)}\,x^3-ax^2+6x-a} \\
x^3-ax^2 \\
-\quad + \\
\hline
6x-a \\
6x-6a \\
-\quad + \\
\hline
5a \\
\hline
\end{array}
$$

Therefore, when $x^3 - ax^2 + 6x - a$ is divided by $x - a$, the remainder obtained is $5a$.

Question 3:

Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.

Solution 3:

Let us divide $(3x^3 + 7x)$ by $(7 + 3x)$.

By long division, we get

$$
\begin{array}{r|l}
 & x^2 - \frac{7}{3}x + \frac{70}{9} \\
\hline
3x+7 & 3x^3 + 0x^2 + 7x \\
 & 3x^3 + 7x^2 \\
 & - \quad - \\
\hline
 & -7x^2 + 7x \\
 & -7x^2 - \frac{49x}{3} \\
 & + \quad + \\
\hline
 & \frac{70x}{3} \\
 & \frac{70x}{3} + \frac{490}{9} \\
 & - \quad - \\
\hline
 & -\frac{490}{9} \\
\hline
\end{array}
$$

The remainder is not zero,

Therefore, $7 + 3x$ is not a factor of $3x^3 + 7x$.